

A study of charm hadron production in e^+e^- annihilation

Novoselov A.A.*

Institute for High Energy Physics, Protvino, Russia and

Moscow Institute of Physics and Technology, Dolgoprudny, Russia

The processes of $D^{(*)}$ -mesons and Λ_c -baryons production in e^+e^- -annihilation at 10.58 GeV and 91.18 GeV energies are concerned. At the 10.58 GeV energy the production of charmed particles via the B -mesons decays is also concerned. Scaling violation of the fragmentation functions is calculated at the NLL-accuracy. Nonperturbative fragmentation functions are retrieved from the experimental data of B -factories and are approximated by simple analytic expressions. It is proved, that the difference between nonperturbative fragmentation functions of mesons and baryons can be easily explained by quark counting.

I. INTRODUCTION

The production of heavy quarks (charm and bottom) in the high-energy collisions and their subsequent hadronization are important processes in particle physics. Apart from being of interest per se, these processes are significant for some practical purposes. For instance, a light Standard Model Higgs boson is expected to decay preferably into heavy quark-antiquark pairs, which then fragment into heavy hadrons. The direct production of heavy quarks would, thereby, be the main background process. Understanding of hadronization processes is also important for the precise determination of the mass of the decaying particle.

Finite heavy-quark mass m_Q , providing a natural infrared cutoff, allows to carry out the calculations to a large extent using perturbative QCD. Nonetheless, differential momentum distributions of the heavy hadrons produced in high-energy collisions are sensitive to the large logarithms $\ln s/m_Q^2$, where s is the center-of-mass energy squared. Since for modern colliders $\sqrt{s} \gg m_Q$, these logarithms threaten the convergence of perturbation expansions. Fortunately, it is possible to show that up to the power corrections in m_Q^2/s the cross

*Electronic address: Alexey.Novoselov@cern.ch

section factorizes into mass-independent hard partonic cross sections convoluted with so-called fragmentation functions describing the probability for the parton to fragment into a particular heavy hadron [1, 2].

Let us consider the inclusive production of a heavy hadron H via the decay of a vector boson $V = \gamma^*, Z^0$ produced in the e^+e^- annihilation:

$$e^+e^- \rightarrow V(q) \rightarrow H(p_H) + X. \quad (1)$$

It is convenient to introduce the scaling variable x for expressing the heavy hadron energy in the center-of-mass frame,

$$x = \frac{2p_H \cdot q}{q^2} \stackrel{\text{C.M.}}{=} \frac{2E_H}{\sqrt{s}}. \quad (2)$$

Experimental data are often presented in terms of the scaled momentum $x_p = |\vec{p}_H|/\sqrt{s/4 - m_H^2}$. But due to large energy of interaction both scaling variables are almost indistinguishable.

As stated above, at leading power in m_Q^2/s the differential production cross section can to all orders of perturbation theory be factorized as

$$\frac{d\sigma_H}{dx} = \sum_a \frac{d\hat{\sigma}_a}{dx}(x, \sqrt{s}) \otimes D_{a/H}(x, m_Q, \mu), \quad (3)$$

where $d\hat{\sigma}_a/dx$ is the cross section for producing a massless parton a with the scaled energy x after subtracting the collinear singularity in the $\overline{\text{MS}}$ factorization scheme, and fragmentation function $D_{a/H}$ gives the probability for a parton a to fragment into a heavy hadron H carrying a fraction x of the parton's momentum. $D_{a/H}$ also depend on the factorization scheme, but the convolution of the two term is not, so that the physical cross section is prescription independent [1].

The factorization formula (3) separates the dependence on the heavy-quark mass m_Q from the dependence on the center-of-mass energy which is contained in the partonic cross sections. These hard cross sections can be calculated in the massless approximation. All dependence on the resulting hadron resides in the fragmentation functions which are process-independent non-perturbative quantities. Like the parton distribution functions (PDF's) fragmentation functions must be measured at some scale and their values at any other scale can be obtained by solving the DGLAP evolution equations [3–5].

The fragmentation functions incorporate long-distance, non-perturbative physics of the hadronization process in which observed hadrons are formed from the partons. For proper

understanding of hadronic uncertainties one needs to separate short $p \sim q$ and long $p \sim \Lambda_{\text{QCD}}$ distance effects. The most popular approach is to factorize the fragmentation function into perturbative and non-perturbative components: $D_{a/H} = D_{a/H}^{\text{pert}} \otimes D_{Q/H}^{np}$ [6–8]. The first component is identified with the so-called perturbative fragmentation function, $D_{a/H}^{\text{pert}}(x, m_Q, \mu) = D_{a/Q}(x, m_Q, \mu)$ while for the non-perturbative component a model such as Kartvelishvili et al. [9, 10] or Peterson et al. [11] is adopted.

Perturbative fragmentation functions $D_{a/Q}$ describe the probabilities for partons a to fragment into an on-shell heavy quark Q . They are relevant for the discussion of inclusive heavy-quark production, where one sums over all possible hadron states H containing heavy quark Q . Quark-hadron duality suggests that

$$\sum_H D_{a/H}(x, m_Q, \mu) = D_{a/Q}(x, m_Q, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_Q(1-x)}\right). \quad (4)$$

However, such a relation can be expected to hold only if x is not too close to 1, so that the scale $m_Q(1-x)$ is in the short-distance regime [12]. It is important to mention, that such an ansatz spoils the proper factorization of short- and long-distance contributions. For $x \rightarrow 1$ perturbative fragmentation function itself contains long-distance contributions from logarithms of momentum scales of order $m_Q(1-x) \sim \Lambda_{\text{QCD}}$. Such logarithms are responsible for soft gluon emission and are not controllable in the perturbation theory. The previous attempts to resum the $\ln^n(1-x)$ terms in the fragmentation functions have thus led to unphysical negative values in the $x \rightarrow 1$ region [13]. As a matter of fact, the Landau singularity of the perturbative coupling at small transverse momenta leads to a branch-point in the resummed expression and produces the negative behavior at $1-x \sim \Lambda_{\text{QCD}}/m_Q$. Soft-gluon resummation therefore suggests that the non-perturbative phenomena become dominant when $x > x_{br} \approx 1 - \Lambda_{\text{QCD}}/m_Q$. The invariant mass of heavy quark and soft gluons emitted can be estimated as $m_Q(1 + (1 - x_{br})/x_{br}) \approx m_Q/(1 - \Lambda_{\text{QCD}}/m_Q) \approx m_Q + \Lambda_{\text{QCD}}$. It means that soft gluons revealing themselves at Λ_{QCD} scale can play appreciable role in the hadronization process.

To incorporate non-perturbative hadronization effects into the heavy-quark fragmentation functions is the main objective of this work. Thus, let us not to resum to all orders of perturbation theory the long-distance terms in the perturbative contribution. The motivation against resummation was adduced above. The non-perturbative fragmentation function will be numerically retrieved from experimental data. Being retrieved from the B -factories

data on the D^* production at the $\Upsilon(4S)$ energy, this function will allow to describe the ALEPH data at Z -boson peak with the reasonable precision. Apart from testing evolution of the D^* fragmentation the difference between meson and baryon fragmentation will be studied. It will be shown that in the $x \rightarrow 1$ region this difference is in agreement with the premises of Kartvelishvili et al. model.

II. THEORETICAL PRELIMINARIES

A. Perturbative fragmentation function and QCD evolution

With the use of factorization relation for fragmentation function and the assumption that $D_{a/H}^{\text{pert}}(x, m_Q, \mu) = D_{a/Q}(x, m_Q, \mu)$ eq. (3) can be rewritten as follows:

$$\begin{aligned} \frac{d\sigma_H}{dx}(x, \sqrt{s}, m_Q) &= \sum_a \frac{d\hat{\sigma}_a}{dx}(x, \sqrt{s}) \otimes D_{a/Q}^{\text{pert}}(x, m_Q, \mu) \otimes D_{Q/H}^{\text{np}}(x) = \\ &= \frac{d\sigma_Q}{dx}(x, \sqrt{s}, m_Q) \otimes D_{Q/H}^{\text{np}}(x), \end{aligned} \quad (5)$$

where $d\sigma_Q/dx$ is the heavy quark differential inclusive cross section.

The $\overline{\text{MS}}$ fragmentation functions $D_{a/Q}$ obey the DGLAP evolution equations

$$\frac{dD_{a/Q}}{d\ln\mu^2}(x, m_Q, \mu) = \sum_b \int_x^1 \frac{dz}{z} P_{ba}\left(\frac{x}{z}, \alpha_s(\mu)\right) D_{b/Q}(z, m_Q, \mu). \quad (6)$$

Let us introduce a notation

$$\bar{\alpha}_s(\mu) = \frac{\alpha_s(\mu)}{2\pi}, \quad (7)$$

where the standard two-loop expression for $\alpha_s(\mu)$ is used. Then the perturbative expansion for the Altarelli-Parisi splitting functions P_{ba} would have the following form:

$$P_{ba}(x, \bar{\alpha}_s(\mu)) = \bar{\alpha}_s(\mu) P_{ba}^{(0)}(x) + \bar{\alpha}_s^2(\mu) P_{ba}^{(1)}(x) + \mathcal{O}(\bar{\alpha}_s^3), \quad (8)$$

where the $P_{ba}^{(0)}$ are [5]

$$\begin{aligned} P_{QQ}^{(0)}(x) &= C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right], \\ P_{gg}^{(0)}(x) &= 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \left(\frac{11}{12} - \frac{n_f T_F}{3C_A} \right) \delta(1-x) \right], \\ P_{gQ}^{(0)}(x) &= C_F \frac{1+(1-x)^2}{x}, \\ P_{Qg}^{(0)}(x) &= T_F \left[x^2 + (1-x)^2 \right], \end{aligned} \quad (9)$$

The NLO splitting functions $P_{ji}^{(1)}$ (needed to achieve NLL accuracy) have been computed in [14–18] and are too lengthy to be replicated here.

The initial conditions for the $\overline{\text{MS}}$ fragmentation functions were first obtained at the NLO level in [1]. They are given by

$$\begin{aligned} D_{Q/Q}^{\text{ini}}(x, m_Q, \mu_0) &= \delta(1-x) + \bar{\alpha}_s(\mu_0) d_Q^{(1)}(x, m_Q, \mu_0) + \mathcal{O}(\bar{\alpha}_s^2), \\ D_{g/Q}^{\text{ini}}(x, m_Q, \mu_0) &= \bar{\alpha}_s(\mu_0) d_g^{(1)}(x, m_Q, \mu_0) + \mathcal{O}(\bar{\alpha}_s^2), \end{aligned} \quad (10)$$

(other $D_{a/Q}$ are of order α_s^2), where

$$\begin{aligned} d_Q^{(1)}(x, m_Q, \mu_0) &= C_F \left[\frac{1+x^2}{1-x} \left(\ln \frac{\mu_0^2}{m_Q^2} - 2 \ln(1-x) - 1 \right) \right]_+, \\ d_g^{(1)}(x, m_Q, \mu_0) &= T_F \left[x^2 + (1-x)^2 \right] \ln \frac{\mu_0^2}{m_Q^2}. \end{aligned} \quad (11)$$

Although the sum in expression (5) runs over all types of partons, $D_{g/Q}$ is α_s -suppressed with respect to $D_{Q/Q}$ while other $D_{a/Q}$ being α_s^2 -suppressed. So in the following let us keep only direct component $D_{Q/Q}$ since it is quite sufficient for the purposes of current work. Thus, expanding the convolution, for heavy quark spectrum one obtains

$$\frac{d\sigma_Q}{dx}(x, \sqrt{s}, m_Q, \mu) = \int_x^1 \frac{dz}{z} \frac{d\hat{\sigma}_Q}{dz}(x/z, \sqrt{s}) D_{Q/Q}(z, m_Q, \mu), \quad (12)$$

where NLO expression for the partonic cross section from [19] should be used:

$$\begin{aligned} \frac{d\hat{\sigma}_Q}{dx}(x, \sqrt{s}) &= \delta(1-x) + \bar{\alpha}_s(\mu) a_Q^{(1)}(x, \sqrt{s}), \\ a_Q^{(1)}(x, \sqrt{s}) &= C_F \left[1 + \ln \frac{s}{m^2} \left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) + \right. \\ &\quad + \frac{1}{2} \frac{x^2 - 6x - 2}{(1-x)_+} - \left(\frac{\ln(1-x)}{1-x} \right)_+ (1+x)^2 + \\ &\quad \left. + 2 \frac{1+x^2}{1-x} \ln x + \left(\frac{2}{3} \pi^2 - \frac{5}{2} \right) \delta(1-x) \right]. \end{aligned} \quad (13)$$

The procedure outlined above guarantees that all leading and next-to-leading logarithmic terms of quasi-collinear origin (terms of the form $(\bar{\alpha}_s \log(q^2/m_Q^2))^n$ and $\bar{\alpha}_s(\bar{\alpha}_s \log(q^2/m_Q^2))^n$ respectively) are correctly resummed in the cross section [1].

For the subsequent analysis it is convenient to turn to the Mellin moments of the quantities involved. The Mellin transformation $f(N)$ of function $f(x)$ is defined as

$$f(N) \equiv \int_0^1 dx x^{N-1} f(x). \quad (14)$$

In Mellin space the evolution equations (6) take the simple form

$$\frac{dD_{a/Q}}{d\ln\mu^2}(N, m_Q, \mu) = \sum_b P_{ba}(N, \alpha_s(\mu)) D_{b/Q}(N, m_Q, \mu). \quad (15)$$

This equation at NLO level was solved analytically in [1] and for the direct component $D_{Q/Q}$ one has:

$$\begin{aligned} D_{Q/Q}(N, m_Q, \mu) &= E(N, \mu, \mu_0) D_{Q/Q}^{\text{ini}}(N, m_Q, \mu_0), \\ E(N, \mu, \mu_0) &= \exp \left\{ \ln \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \frac{P_{QQ}^{(0)}(N)}{2\pi b_0} + \right. \\ &\quad \left. + \frac{\alpha_s(\mu_0) - \alpha_s(\mu)}{4\pi^2 b_0} \left[P_{QQ}^{(1)}(N) - \frac{2\pi b_1}{b_0} P_{QQ}^{(0)}(N) \right] \right\}. \end{aligned} \quad (16)$$

Defining

$$\sigma_c(N, \sqrt{s}) \equiv \int_0^1 dx x^{N-1} \frac{d\sigma_c}{dx}(x, \sqrt{s}), \quad (17)$$

one has the following expression for the NLO distribution:

$$\sigma_c(N, \sqrt{s}) = \hat{\sigma}_Q(N, \sqrt{s}) E(N, \mu, \mu_0) D_{Q/Q}^{\text{ini}}(N, \mu_0, m_Q). \quad (18)$$

Both $a_Q^{(1)}$ and $d_Q^{(1)}$ contain terms proportional to the $\alpha_s/(1-x)_+$ and $\alpha_s [\ln(1-x)/(1-x)]_+$, associated to the emission of a soft gluon. These terms give rise to a large- N growth of the corresponding Mellin transforms

$$\begin{aligned} a_Q^{(1)}(N, \sqrt{s}, \mu) &= C_F \left[\ln^2 N + \left(\frac{3}{2} + 2\gamma_E - 2 \ln \frac{s}{\mu^2} \right) \ln N + \alpha_Q + \mathcal{O}(1/N) \right], \\ d_Q^{(1)}(N, \mu_0, m_Q) &= C_F \left[-2 \ln^2 N + 2 \left(\ln \frac{m^2}{\mu_0^2} - 2\gamma_E + 1 \right) \ln N + \delta_Q + \mathcal{O}(1/N) \right]. \end{aligned} \quad (19)$$

Leading $\alpha_s^n \ln^{n+1} N$ and next to leading $\alpha_s^n \ln^n N$ logarithmic contributions were resummed to all orders of perturbation theory in [20]. Opposed to fixed order calculation, which leads to finite and positive fragmentation function at almost all values of x except the $x \rightarrow 1$ region (where the δ -function from the initial condition becomes apparent), NLL resummed result exhibit pathological negative behavior when x approaches 1. The reason is that the Landau singularity of the perturbative QCD coupling at small transverse momenta leads to branch-points in the resummed expression for the initial condition and the coefficient function.

In the initial condition the singularities start at the branch-point

$$N_{\text{ini}}^L = \exp\left(\frac{1}{2b_0\alpha_s(\mu_0)}\right) \simeq \frac{\mu_0}{\Lambda_{\text{QCD}}}, \quad (20)$$

while for the coefficient functions the branch-point is

$$N_q^L = \exp\left(\frac{1}{b_0\alpha_s(\mu)}\right) \simeq \frac{\mu^2}{\Lambda_{\text{QCD}}^2}. \quad (21)$$

The previous attempts to restore the physical behavior of fragmentation functions consisted in introducing a tower of power corrections to N represented by the replacement

$$N \rightarrow N \frac{1 + f/N_{\text{ini}}^L}{1 + f N/N_{\text{ini}}^L}, \quad (22)$$

in the initial condition and

$$N \rightarrow N \frac{1 + f/N_q^L}{1 + f N/N_q^L}, \quad (23)$$

in the coefficient function. It is easy to see that f being more or equal to 1 unphysical region is unreachable. But it is important to keep in mind that there is no rigorous justification for the replacements (22), (23).

B. Non-perturbative fragmentation function

The most popular parameterizations for the non-perturbative fragmentation functions are Peterson et al. [11] and Kartvelishvili et al. (KLP) [9, 10]. The former has a form of the heavy quark propagator and does not depend on the hadron produced. Thus let us concentrate on the latter one. It is based on the Gribov-Lipatov “reciprocity relation” between $D_{Q/H}^{\text{np}}$ and the distribution function of quark Q in hadron H [4]:

$$D_{Q/H}^{\text{np}}(z) \stackrel{z \rightarrow 1}{\simeq} f_H^Q(z), \quad (24)$$

where $z = p_H/p_c$ is the hadron momentum fraction with the respect to the heavy quark momentum. The expression for $f_{D^*}^c$ was found out in [26] on the basis of Kuti-Weisskopf model. The parametrization obtained is significantly related to the Regge trajectory parameters of the $Q\bar{Q}$ -system and has the following form:

$$f_{D^*}^c(x) = \frac{\Gamma(2 + \gamma_M - \alpha_Q - \alpha_q)}{\Gamma(1 - \alpha_Q)\Gamma(1 + \gamma_M - \alpha_q)} x^{-\alpha_Q} (1 - x)^{\gamma_M - \alpha_q}, \quad (25)$$

where $\alpha_q = 1/2$ is the intercept of the light quarks trajectory ρ, ω, f, A_2 , α_Q is the intercept of the leading trajectory for the $Q\bar{Q}$ -system and γ_H is a parameter determining the behavior of the distribution function for $x \rightarrow 1$. Its value originates from the $q^{(-2)}$ diminution of the form-factor as it is known that $q^{(-2k)}$ diminution of form-factor leads to the $(1-x)^{2k-1}$ behavior of the distribution function at large x . Assuming the universality of the sea quarks distribution in all mesons one gets $\gamma_M = 3/2$. In much the same way for Λ_C -baryons the following expression was found out:

$$f_{\Lambda_c}^c(x) = \frac{\Gamma(3 + \gamma_B - \alpha_Q - 2\alpha_q)}{\Gamma(1 - \alpha_Q)\Gamma(1 + \gamma_B - 2\alpha_q)} x^{-\alpha_c} (1-x)^{1+\gamma_B-2\alpha_q}, \quad (26)$$

where $\gamma_B = 3$ and the factor 2 is related to the number of valent light quarks in baryon.

Gribov-Lipatov “reciprocity” relation with known distribution functions (25), (26) determine the behaviour of the fragmentation functions in the $x \rightarrow 1$ limit. For the small values of x the condition for the fragmentational approach to be valid ($p \gg m_Q$) is not satisfied. Nonetheless, if at small x values the production of heavy hadrons mainly depend on their wave functions, then coinciding $x \rightarrow 1$ asymptotes of (25) and (26) prognosticate similar behaviour of the momentum distributions of mesons and baryons in this region.

There is still some uncertainty in the value of α_c . Theoretical investigations [27] based on Regge trajectory systematics result in the value for α_c in the range between -2.0 and -3.5 . It is slightly more than the value of $\alpha_c \approx -3 \div -4$, obtained in [28] with the use of the QCD sum rules. Another way to determine α_c by the value of the heavy quarkonia wave function in the center point leads to the value -3.5 ± 0.6 [29].

III. CHARM HADRONS DATA FITS NEAR THE $\Upsilon(4S)$

High quality data on the charmed hadron production is provided by BELLE, BABAR and CLEO collaborations [21–23]. One thus has the opportunity to perform a more accurate fit for the non-perturbative initial conditions. Furthermore, it gives us the possibility to test the evolution of the fragmentation function from the center-of-mass energies of 10.6 to 91.2 GeV, using charm data from the LEP experiments [24, 25].

For the perturbative component of fragmentation function we use expression (18) with the NLO initial condition (10), the NLO partonic cross section (13) and the NLL evolution (16) [33]. As stated above we do not perform NLL Sudakov resummation for the initial conditions

and coefficient functions retaining these long-distance contributions for the non-perturbative component.

Several parameters enter the perturbative calculations. First of all these are the initial μ_0 and the final μ evolution scales which allow variation by a factor of order of 2 around m_c and \sqrt{s} respectively. We set them to be $\mu_0 = 2m_c$ and $\mu = \sqrt{s}/2$ as this values allow to perform the most successful fitting. The center-of-mass energy \sqrt{s} is equal to $m_{\Upsilon(4S)} = 10.58$ GeV. We shall use the pole mass for charm quark and fix it to be $m_c = 1.6$ GeV. As the b -quark mass lies between μ_0 and μ it creates a threshold on the different sides of which different number of active flavours enter the evolution operator. We set $m_b = 5.0$ GeV. Experimental value of $\alpha_s(m_Z) = 0.119$ points to the value of Λ_{QCD} equal 0.226 GeV.

There are several ways to retrieve the non-perturbative component. DGLAP equations allow to perform the evolution from a larger scale to a lower one as well as in the opposite direction. But then one needs to extract the non-perturbative function from its convolution with the partonic cross section and the perturbative initial condition. This procedure is connected with the inverse Mellin transform of the moments of experimental data divided by the moments of the the partonic cross section and the perturbative initial condition. Such calculation performed numerically faces problems with the integral's convergence developing into unphysical negative values of the non-perturbative function in the low x region. It was tested that resulting non-perturbative function obtained by such way does not permit to successively reproduce the experimental data by the reverse procedure.

Anther method, proposed in this work, is to represent the function required as a linear combination of functions which have a simple analytic form of Mellin transform. Further, expansion coefficients can be determined by fitting to experimental data in the Mellin space as well as in the x -space. The most general choice is to retrieve the non-perturbative fragmentation function bin-by-bin, choosing the number of bins n to be not more than the number of experimental points to avoid overdetermined system for the coefficients. So, let us define

$$\widetilde{D}^{\text{np}}(z) = \sum_{i=1}^n c_i \Theta\left(z - \frac{i-1}{n}\right) \Theta\left(\frac{i}{n} - z\right). \quad (27)$$

The corresponding Mellin transform is

$$\widetilde{D}^{\text{np}}(N) = \sum_{i=1}^n c_i \int_{\frac{i-1}{n}}^{\frac{i}{n}} z^{N-1} dz = \sum_{i=1}^n c_i \frac{(i/n)^N - ((i-1)/n)^N}{N}. \quad (28)$$

The approximate number of points in the BELLE and BABAR data is 50, in the CLEO

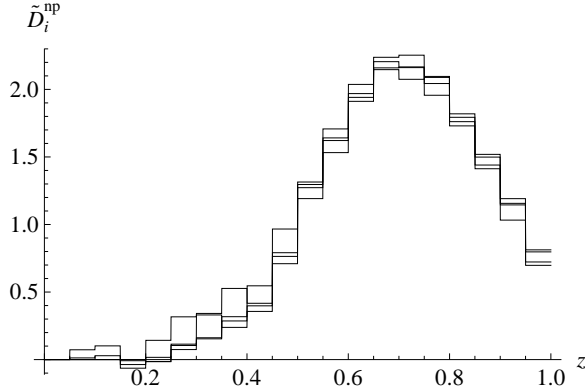


FIG. 1: Nonperturbative fragmentation functions of the D^* -mesons extracted from several datasets.

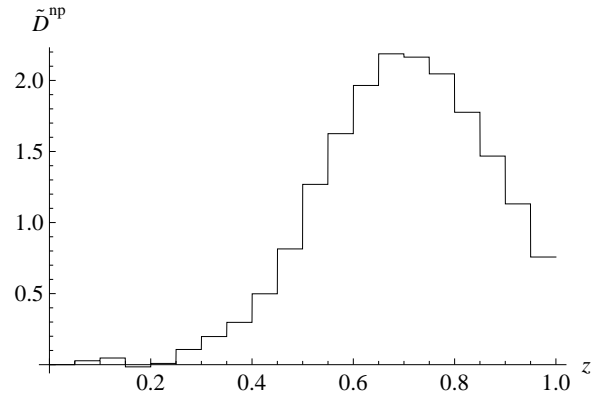


FIG. 2: Averaged nonperturbative fragmentation function of the D^* -mesons.

data — 20. Thus to take 20 bins for the non-perturbative function required seems to be a reasonable choice.

Fit to the experimental data in the N -space can be performed as well as in the x -space. The c_i coefficients obtained by both techniques are in a good agreement. There are 4 datasets for D^* production at $\Upsilon(4S)$ energies. Two of them presented by BELLE and two by CLEO, they regard to the D^{*+} and D^{*0} production. The non-perturbative fragmentation functions extracted from them are plotted in Fig. 1. As we assume that there is no difference between the D^{*+} and D^{*0} fragmentation these functions can be averaged to get the D^* non-perturbative fragmentation function plotted in Fig. 2. The weights in this average were selected proportional to the statistics gathered for each data-set.

The same procedure was carried out for the Λ_C -baryon production at BABAR and BELLE. The corresponding plots for the non-perturbative fragmentation functions are presented in Fig. 3 and 4.

The mesonic and baryonic fragmentation functions obtained in such way do not reveal significantly different behaviour at $z \lesssim 0.5$ (Fig. 5). Furthermore, best fits by $c \cdot x^{-\alpha}$ function in the $x < 0.5$ region for both cases result in close values of α . These values being equal to -3.7 and -3.8 are in a pretty good agreement with the previously mentioned predictions for α_c . What concerns the $z \rightarrow 1$ behaviour, the difference in it is in agreement with the Gribov-Lipatov “reciprocity” relation.

For further phenomenological analysis it is convenient to find some simple expression

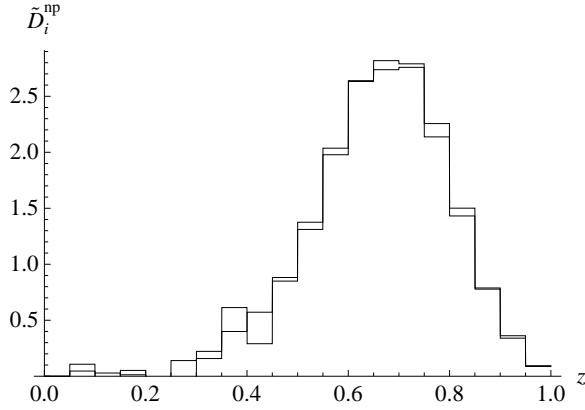


FIG. 3: Nonperturbative fragmentation functions of the Λ_C -baryons extracted from several datasets.

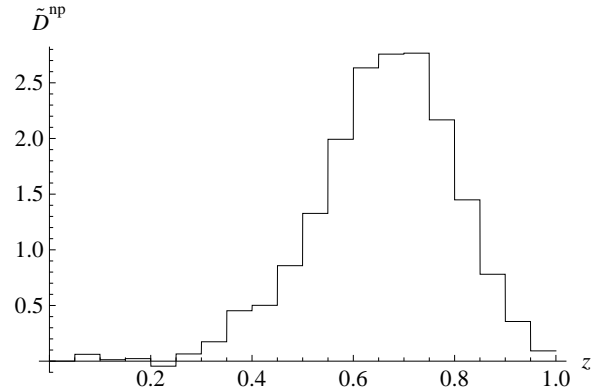


FIG. 4: Averaged nonperturbative fragmentation function of the Λ_C -baryons.

approximating the numeric data obtained:

$$\widehat{D}_c^{D^*}(z) = 20.1z^{3.7}(1-z) + 2.77 \cdot 10^3 z^{13}(1-z)^7, \quad (29)$$

$$\widehat{D}_c^{\Lambda_c}(z) = 72.9z^{3.7}(1-z)^5 + 2.93 \cdot 10^4 z^{10}(1-z)^5 + 10^3 z^{10}(1-z)^3. \quad (30)$$

Each of them has the same $z \rightarrow 0$ and $z \rightarrow 1$ asymptotes as the corresponding KLP function.

IV. CHARMED HADRON PRODUCTION IN ANNIHILATION PROCESSES

According to the formula (5) moments of the momentum spectrum of the particles produced are equal to the product of moments of the differential partonic cross-section (18) and moments of the corresponding non-perturbative fragmentation function. To return to the x variable the inverse Mellin-transform should be performed by integrating over the vertical line in a complex plane:

$$\frac{d\sigma_H}{dx}(x, \sqrt{s}) = \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{dN}{2\pi i} x^{-N} \sigma_H(N, \sqrt{s}). \quad (31)$$

As in a fixed-order calculation the Landau pole does not appear it is possible to use any positive value of γ . Coincident results obtained at different values of γ prove the independence on its value.

Non-perturbative functions (29) and (30) allow to reproduce the experimental data from B -factories with good precision, see Fig. 6, 7 and 8.

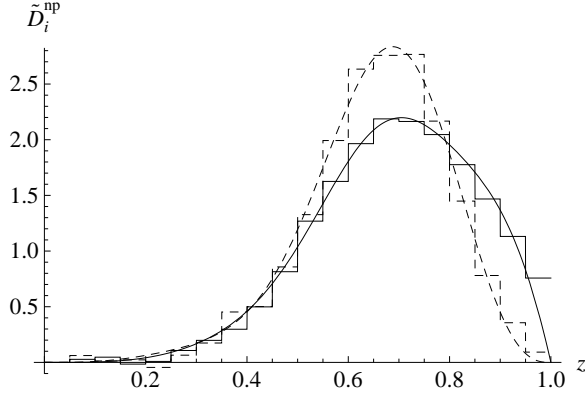


FIG. 5: Nonperturbative fragmentation functions of D^* -mesons and Λ_C -baryons together with the approximative expressions (29) and (30).

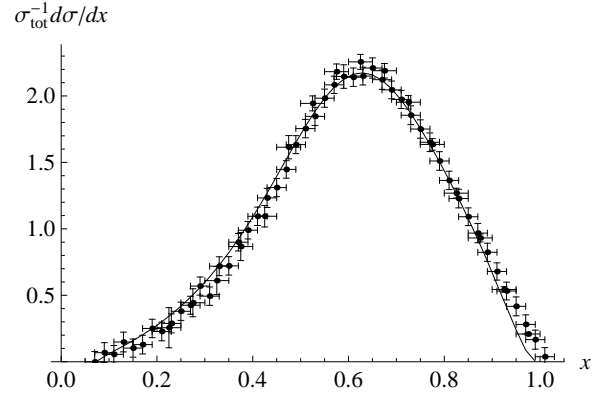


FIG. 6: D^{*+} -meson momentum distribution from the c -quark fragmentation at $\sqrt{s} = 10.58$ GeV energy together with the Belle and CLEO experimental data.

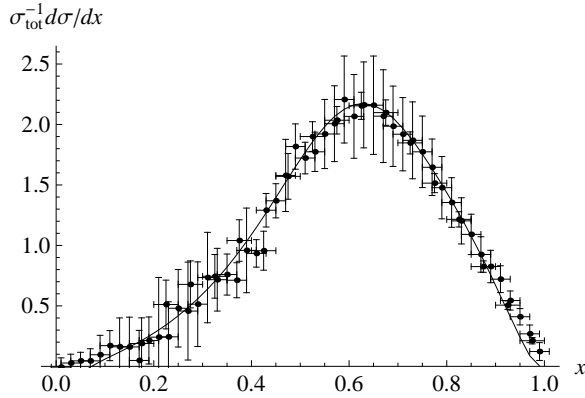


FIG. 7: D^{*0} -meson momentum distribution from the c -quark fragmentation at $\sqrt{s} = 10.58$ GeV energy together with the Belle and CLEO experimental data.

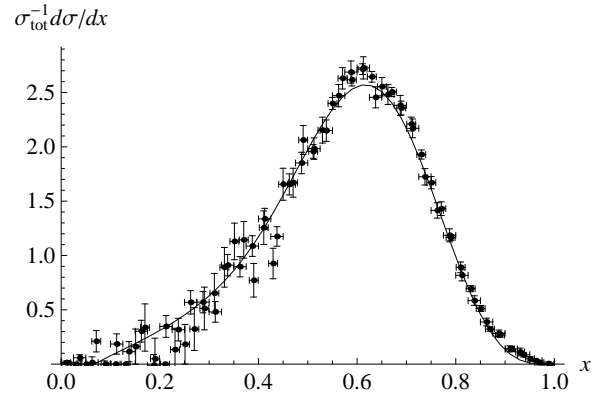


FIG. 8: Λ_C -baryon momentum distribution from the c -quark fragmentation at $\sqrt{s} = 10.58$ GeV energy together with the BaBar and Belle experimental data.

Evolution to the scale $\sqrt{s}/2 = 45.6$ GeV and non-perturbative expression (29) are used to obtain the momentum distribution of D^* -mesons at the Z -boson peak. The predicted spectrum together with the ALEPH and OPAL data is presented in Fig. 9. These distributions coincide with the reasonable precision. As the only difference in calculations for the 10 and 90 GeV energies was in the final evolution scale, one can state that factorization relation is valid in this energy range.

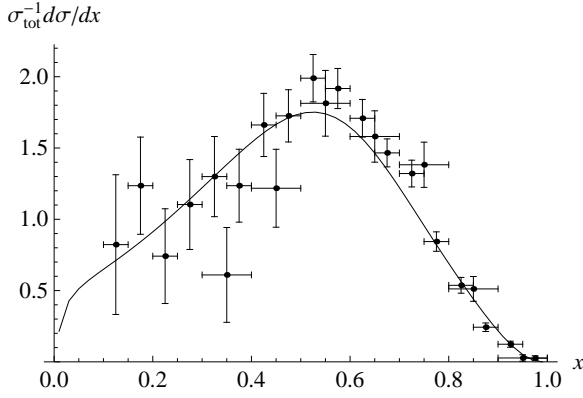


FIG. 9: D^{*+} -meson momentum distribution from the c -quark fragmentation at $\sqrt{s} = 91.18$ GeV energy together with the ALEPH and OPAL experimental data.

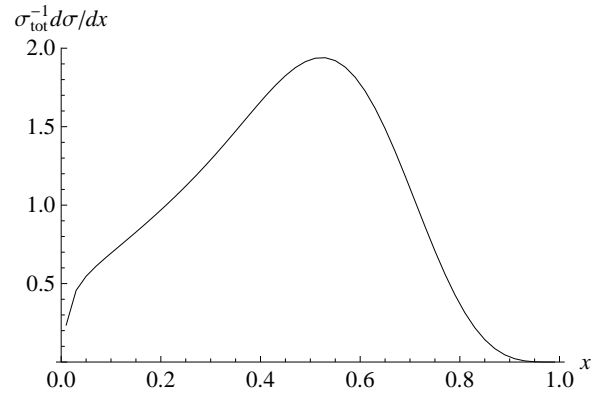


FIG. 10: Predicted Λ_c -baryon momentum distribution from the c -quark fragmentation at $\sqrt{s} = 91.18$ GeV energy.

For the Λ_c production at 91.2 GeV energy the same evolution in the perturbative component is used. The non-perturbative effects are described by the expression (30). Unfortunately there is no experimental data on the Λ_c production at Z -boson peak. Our prediction for the Λ_c momentum distribution is presented in Fig. 10.

A considerable part of D 's is produced indirectly through D^* decays. The simple spin-state counting estimation $(2J + 1)$ leads to the factor 3 enhancement for D^* production. The experimental value obtained at Z -boson peak amounts to only 1.4 [31]. We will use experimental value and assume that D^{*+} , D^{*-} , D^{*0} and \bar{D}^{*0} [34] are produced with equal probabilities.

Following the approach of [30] we assume that the D meson non-perturbative fragmentation function is the sum of a direct component, which is isospin invariant plus the component arising from the D^* decay.

The decay $D^* \rightarrow D\pi$ takes place very close to the threshold. The momentum of D -meson in the D^* -meson rest system

$$p' = \sqrt{\left(\frac{m_{D^*}^2 + m_D^2 - m_\pi^2}{2m_{D^*}}\right)^2 - m_D^2} = 16 \quad (32)$$

is sufficiently small to be neglected. Thus D has the same velocity as the D^* , and their momenta are thus proportional to their masses. So the component of the D -meson fragmen-

tation function arising from $D^* \rightarrow D\pi$ decay is given by

$$\tilde{D}^{D\pi}(z) = D_c^{D^*} \left(z \frac{m_{D^*}}{m_D} \right) \theta \left(1 - z \frac{m_{D^*}}{m_D} \right) \frac{m_{D^*}}{m_D}, \quad (33)$$

where $D_c^{D^*}(z)$ is the non-perturbative fragmentation function of D^* -meson (29). The integral of the expression (33) equals 1, so it should enter the D -meson fragmentation function with the weight proportional to the $D^* \rightarrow D\pi$ decay probability and probability of the D^* production.

What concerns the $D^* \rightarrow D\gamma$ decay, momentum of the D in the D^* frame is non-negligible:

$$p' = \frac{m_{D^*}^2 - m_D^2}{2m_{D^*}} = 135. \quad (34)$$

The D momentum in the laboratory frame is given by a Lorentz boost

$$p = \gamma(p' \cos \theta + \beta \epsilon'), \quad (35)$$

where β is the velocity of the D^* -meson, $\gamma = 1/\sqrt{1-\beta^2}$, $\epsilon' = (m_{D^*}^2 + m_D^2)/(2m_{D^*})$ — energy of the D -meson in the D^* rest frame and θ — its decay angle with respect to the D^* direction. Denoting momentum and energy of the D^* in laboratory frame by p^* and ϵ^* respectively one obtains

$$\gamma = \frac{\epsilon^*}{m_{D^*}}, \quad \beta = \frac{p^*}{\gamma m_{D^*}}. \quad (36)$$

Introducing variables

$$\begin{aligned} z &= \frac{p}{p_{\max}^D} \equiv \frac{p}{\sqrt{s/4 - m_D^2}}, \\ z^* &= \frac{p^*}{p_{\max}^{D^*}} \equiv \frac{p^*}{\sqrt{s/4 - m_{D^*}^2}}, \end{aligned} \quad (37)$$

contribution of the $D^* \rightarrow D\gamma$ decay to the D production can be written down as

$$\tilde{D}^{D\gamma}(z) = \int_0^1 dz^* \int_{-1}^1 \frac{d \cos \theta}{2} D_c^{D^*}(z^*) \delta \left(z - \gamma \frac{p' \cos \theta + \beta \epsilon'}{p_{\max}} \right). \quad (38)$$

As in the previous case the integral of this expression is normalized to unity.

The branching ratios involved are [32]:

$$Br_{D^{*+} \rightarrow D^0 \pi^+} = 67.7 \pm 0.5, \%$$

$$\begin{aligned}
Br_{D^{*+} \rightarrow D^+ \pi^0} &= 30.7 \pm 0.5, \% \\
Br_{D^{*+} \rightarrow D^+ \gamma} &= 1.6 \pm 0.4, \% \\
Br_{D^{*0} \rightarrow D^0 \pi^0} &= 61.9 \pm 2.9, \% \\
Br_{D^{*0} \rightarrow D^0 \gamma} &= 38.1 \pm 2.9, \%
\end{aligned} \tag{39}$$

Finally non-perturbative fragmentation functions of D -mesons can be written down as follows:

$$\begin{aligned}
\tilde{D}_c^{D^+}(z) &= n^{D^+} (D_c^D(z) + c [Br_{D^{*+} \rightarrow D^+ \gamma} \tilde{D}^{D\gamma}(z) + \\
&+ Br_{D^{*+} \rightarrow D^+ \pi^0} \tilde{D}^{D\pi}(z)])
\end{aligned} \tag{40}$$

and

$$\begin{aligned}
\tilde{D}_c^{D^0}(z) &= n^{D^0} (D_c^D(z) + c [Br_{D^{*0} \rightarrow D^0 \gamma} \tilde{D}^{D\gamma}(z) + \\
&+ (Br_{D^{*+} \rightarrow D^0 \pi^+} + Br_{D^{*0} \rightarrow D^0 \pi^0}) \tilde{D}^{D\pi}(z)])
\end{aligned} \tag{41}$$

where $c = 1.4$ — ratio of probabilities to fragment into D^* and D -mesons, coefficients n^{D^+} and n^{D^0} provide normalization of fragmentation functions to unity:

$$\begin{aligned}
n^{D^+} &= (1 + c(Br_{D^{*+} \rightarrow D^+ \gamma} + Br_{D^{*+} \rightarrow D^+ \pi^0}))^{-1}, \\
n^{D^0} &= (1 + c(Br_{D^{*0} \rightarrow D^0 \gamma} + Br_{D^{*+} \rightarrow D^0 \pi^+} + Br_{D^{*0} \rightarrow D^0 \pi^0}))^{-1}.
\end{aligned} \tag{42}$$

In order to obtain momentum spectra of D -mesons expression (31) is used. The result for D^+ -mesons is presented in Fig. 11, for D^0 -mesons — in Fig. 12. Both distributions are in a good agreement with experimental data. The D^0 spectrum is slightly softer then the D^+ one because of the larger probability of the $D^* \rightarrow D^0 X$ decay.

V. CHARMED HADRON PRODUCTION IN B -MESON DECAYS

Let us now consider charmed particles production in the b -quark decays. The energy of 10.58 GeV corresponds to the $\Upsilon(4S)$ resonance which decays into a $B\bar{B}$ -pair with almost unitary probability. B -mesons from such decays are nearly at rest as their mass $m_B = 5.28 \text{ GeV} \simeq \sqrt{s}/2$. Neglecting b -quark motion within the meson the c -quark spectrum from B decay can be easily found. The charmed hadron spectrum can then be written down as

$$\frac{d\sigma_H}{dx}(x) = \int_x^1 \frac{dz}{z} \left(\frac{d\sigma_{b \rightarrow c}}{dz}(z) \right) D_c^{\text{np}}\left(\frac{x}{z}\right), \tag{43}$$

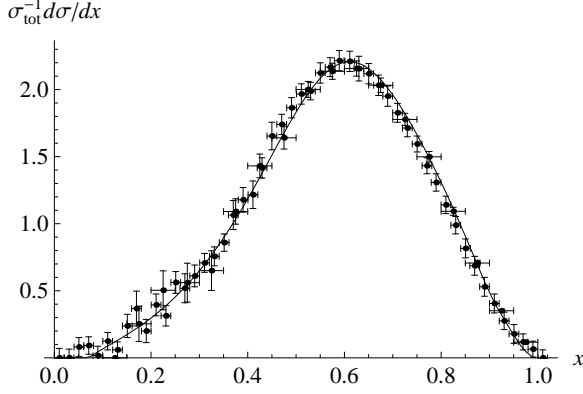


FIG. 11: D^+ -meson momentum distribution from the c -quark fragmentation at $\sqrt{s} = 10.58$ GeV energy together with the Belle and CLEO experimental data.

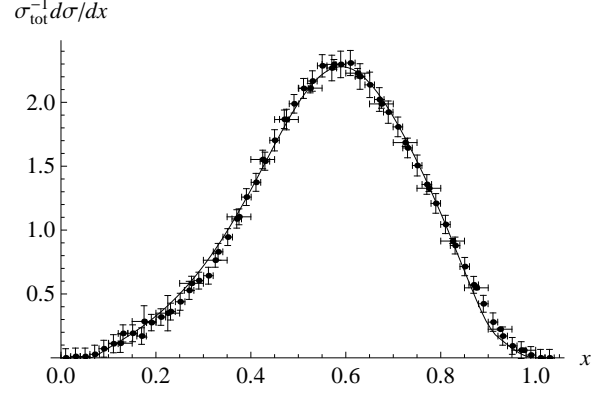


FIG. 12: D^0 -meson momentum distribution from the c -quark fragmentation at $\sqrt{s} = 10.58$ GeV energy together with the Belle and CLEO experimental data.

where $d\sigma_{b \rightarrow c}/dz$ is the c -quark spectrum obtained by analysis of weak decays $b \rightarrow c + l\nu_l$, $b \rightarrow c + q\bar{q}$, $b \rightarrow c + \bar{c}s$ and $\bar{b} \rightarrow \bar{c} + c\bar{s}$.

We neglect here any perturbative fragmentation function as at such low energy it should not sufficiently differ from the δ -function. The non-perturbative component of D^* production is described by the expression (29).

In experimental data value $x = 1$ corresponds to the largest possible momentum of the D^* -meson produced at 10.58 GeV energy:

$$p_{\max}^{D^*} = \sqrt{s/4 - m_{D^*}^2} = 4.88 \text{ GeV} \quad (44)$$

while the largest possible momentum of the D^* -meson from the B decay is equal to the largest c -quark momentum

$$p_{\max}^c = \sqrt{\frac{(m_b^2 + m_c^2)^2}{4m_b^2} - m_c^2} = 2.24 \text{ GeV}. \quad (45)$$

This value actually coincides with those calculated via the hadron (not parton) masses:

$$p_{\max}^{B \rightarrow D^*} = \sqrt{\frac{(m_B^2 + m_{D^*}^2)^2}{4m_B^2} - m_{D^*}^2} = 2.26 \text{ GeV}. \quad (46)$$

Thus the momentum distribution of D -mesons from the B decays is located in the region $x < p_{\max}^c/p_{\max}^{D^*} = 0.46$. The distribution concerned and the experimental data are plotted in

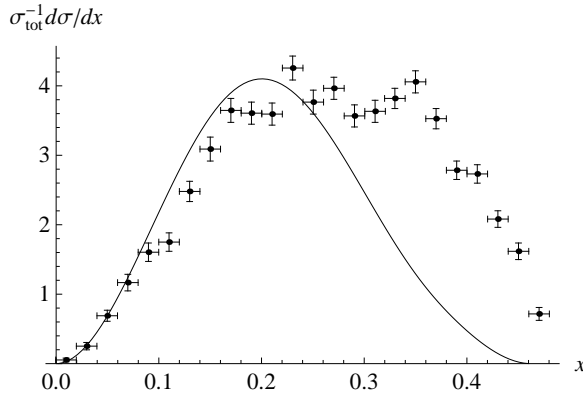


FIG. 13: D^* -meson momentum distribution from the b -quark fragmentation at $\sqrt{s} = 10.58$ GeV energy together with the Belle experimental data.

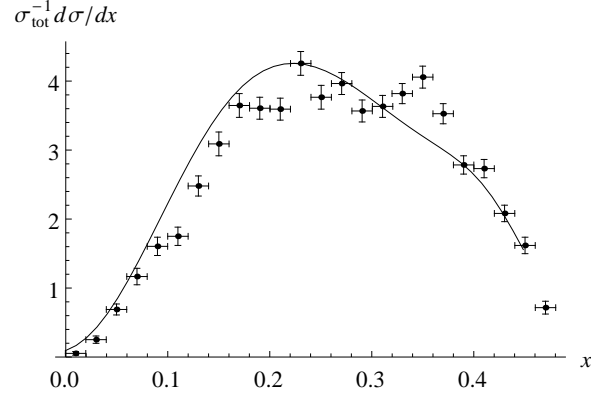


FIG. 14: D^* -meson momentum distribution from the b -quark fragmentation and recombination with the light quark from B -meson at $\sqrt{s} = 10.58$ GeV energy together with the Belle experimental data.

Fig. 13. In this only plot the normalization of the experimental data was changed for points in the low x region to be in agreement with the predictions. In the high x region there is some extra contribution not predicted by the fragmentation model. We suppose that this contribution arises from the interaction of the charm quark with the light valent quark from the B -meson. As this quark has larger momentum than a sea quark the process concerned contributes to the high x region. We assume that the momentum distribution of D^* -mesons produced in this process coincides with the c -quark distribution. This assumption allows to obtain better agreement with the experimental data (Fig. 14).

Let us now proceed to the Λ_C -baryon production. To obtain Λ_C -baryon momentum distribution one should convolute c -quark spectrum with the non-perturbative fragmentation function (30). The largest possible momentum of the Λ_C -baryon produced at 10.58 GeV energy is equal

$$p_{\max}^{\Lambda_c} = \sqrt{s/4 - m_{\Lambda_c}^2} = 4.76. \quad (47)$$

When finding largest Λ_C -baryon momentum from the B decay it is important to mention that baryon production is always accompanied by the any-baryon emergence. The lightest

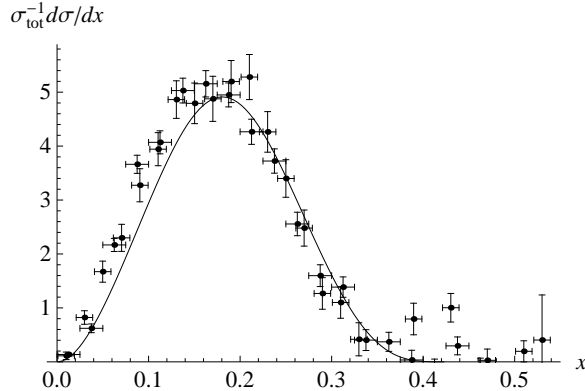


FIG. 15: Λ_C -baryon momentum distribution from the b -quark fragmentation at $\sqrt{s} = 10.58$ GeV energy together with the BaBar and Belle experimental data.

of them is anti-proton. Thus the largest possible Λ_C -baryon momentum equals to

$$p_{\text{max}}^{B \rightarrow \Lambda_c} = \sqrt{\frac{(m_B^2 + m_{\Lambda_c}^2 - m_p^2)^2}{4m_B^2}} - m_{\Lambda_c} = 2.02. \quad (48)$$

This value differs significantly from the p_{max}^c .

Thereby the distribution sought for occupies the region $x < p_{\text{max}}^{B \rightarrow \Lambda_c} / p_{\text{max}}^{\Lambda_c} = 0.42$. It reasonably coincides with the experimental data (Fig. 15). As opposite to the meson production sea ud -diquark capturing is needed for the Λ_C formation. That is why valent quark from the B -meson does not change the prediction of fragmentation approach in case of baryon production.

VI. CONCLUSION

In this article charmed hadron production in the vast range of energies was concerned. During this the energy dependence was contained only in the perturbative component of fragmentation function, for which the NLO-expression was used. Good agreement with the experimental data points to the fulfilment of the factorization assumption in the 10 to 90 GeV energy range.

It is important to mention that non-perturbative fragmentation functions were one and the same for all energies except D^* -mesons production in B decays. It means that separation of non-perturbative phenomena was carried out correctly. Indeed the perturbative part is relevant for charmed quark production only, the non-perturbative — only for its transaction

to the final particle.

The difference in meson and baryon production in the $x \rightarrow 1$ region dues to the different non-perturbative fragmentation functions. The difference in them by-turn is explained by quark-counting. This phenomenon takes place in large x region only, where non-perturbative phenomena is most significant.

In the $x \rightarrow 0$ region non-perturbative fragmentation functions of charmed mesons and baryons coincide. This is relevant to the one and the same behavior of charm quark distribution functions in mesons and baryons in the low- x limit. Moreover, the parameter α which determines the low- x behavior is close to the estimations of the α_c — interception of the charmed Regge trajectory.

At the low energy scale which corresponds to the B -meson mass a discrepancy in the D^* momentum distribution was found. The origin of it is thought to due mainly to the influence of a valent quark from decaying B -meson. Such discrepancy does not occur in Λ_C production which is in a pretty good agreement with the experimental data. The m/\sqrt{s} -corrections to the factorization relation also worth mentioning at the m_B energy scale.

Author would like to thank Prof. Likhoded A.K. and Luchinsky A.V. for useful ideas and discussions. The work was financially supported by Russian Foundation for Basic Research (grant#10-02-00061-a) and grants of the president of Russian Federation MK-140.2009.2 and MK-406.2010.2.

-
- [1] B. Mele and P. Nason, Nucl. Phys. B **361**, 626 (1991).
 - [2] J. C. Collins, Phys. Rev. D **58**, 094002 (1998) [hep-ph/9806259].
 - [3] Y. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977) [Zh. Eksp. Teor. Fiz. **73**, 1216 (1977)].
 - [4] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972) [Yad. Fiz. **15**, 781 (1972)].
 - [5] G. Altarelli and G. Parisi, Nucl. Phys. B **126**, 298 (1977).
 - [6] G. Colangelo and P. Nason, Phys. Lett. B **285**, 167 (1992).
 - [7] M. Cacciari, M. Greco, S. Rolli and A. Tanzini, Phys. Rev. D **55**, 2736 (1997) [hep-ph/9608213].
 - [8] P. Nason and C. Oleari, Nucl. Phys. B **565**, 245 (2000) [hep-ph/9903541].
 - [9] V. G. Kartvelishvili, A. K. Likhoded and V. A. Petrov, Phys. Lett. B **78**, 615 (1978).

- [10] V. G. Kartvelishvili and A. K. Likhoded, Sov. J. Nucl. Phys. **29**, 390 (1979) [Yad. Fiz. **29**, 757 (1979)].
- [11] C. Peterson, D. Schlatter, I. Schmitt and P. M. Zerwas, Phys. Rev. D **27**, 105 (1983).
- [12] M. Neubert, arXiv:0706.2136 [hep-ph].
- [13] M. Cacciari, P. Nason and C. Oleari, JHEP **0604**, 006 (2006) [arXiv:hep-ph/0510032].
- [14] G. Curci, W. Furmanski, and R. Petronzio, *Evolution of parton densities beyond leading order: The nonsinglet case*, Nucl. Phys. **B175** (1980) 27.
- [15] W. Furmanski and R. Petronzio, *Singlet parton densities beyond leading order*, Phys. Lett. **B97** (1980) 437.
- [16] E. G. Floratos, C. Kounnas, and R. Lacaze, *Higher order qcd effects in inclusive annihilation and deep inelastic scattering*, Nucl. Phys. **B192** (1981) 417.
- [17] J. Kalinowski, K. Konishi, P. N. Scharbach, and T. R. Taylor, *Resolving qcd jets beyond leading order: Quark decay probabilities*, Nucl. Phys. **B181** (1981) 253.
- [18] J. Kalinowski, K. Konishi, and T. R. Taylor, *Jet calculus beyond leading logarithms*, Nucl. Phys. **B181** (1981) 221.
- [19] P. Nason and B. R. Webber, *Scaling violation in e^+e^- fragmentation functions: Qcd evolution, hadronization and heavy quark mass effects*, Nucl. Phys. **B421** (1994) 473–517. Erratum-ibid.B480:755,1996.
- [20] M. Cacciari and S. Catani, Nucl. Phys. B **617**, 253 (2001) [arXiv:hep-ph/0107138].
- [21] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **75**, 012003 (2007) [arXiv:hep-ex/0609004].
- [22] R. Seuster *et al.* [Belle Collaboration], Phys. Rev. D **73**, 032002 (2006) [arXiv:hep-ex/0506068].
- [23] M. Artuso *et al.* [CLEO Collaboration], Phys. Rev. D **70**, 112001 (2004) [arXiv:hep-ex/0402040].
- [24] R. Barate *et al.* [ALEPH Collaboration], Eur. Phys. J. C **16**, 597 (2000) [arXiv:hep-ex/9909032].
- [25] K. Ackerstaff *et al.* [OPAL Collaboration], Eur. Phys. J. C **1**, 439 (1998) [arXiv:hep-ex/9708021].
- [26] P. V. Chliapnikov, V. G. Kartvelishvili, V. V. Knyazev and A. K. Likhoded, Nucl. Phys. B **148**, 400 (1979).
- [27] S. S. Gershtein, A. K. Likhoded and A. V. Luchinsky, Phys. Rev. D **74**, 016002 (2006)

- [arXiv:hep-ph/0602048].
- [28] A. Y. Khodjamirian and A. G. Oganesian, Phys. Atom. Nucl. **56**, 1720 (1993) [Yad. Fiz. **56**, 172 (1993)].
- [29] V. G. Kartvelishvili and A. K. Likhoded, Yad. Fiz. **42**, 1306 (1985) [Sov. J. Nucl. Phys. **42**, 823 (1985)].
- [30] M. Cacciari and P. Nason, *Charm cross sections for the tevatron run ii*, *JHEP* **09** (2003) 006, [hep-ph/0306212].
- [31] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **75**, 072002 (2007) [arXiv:hep-ex/0606026].
- [32] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
- [33] Actually an expression analogous to (4.15) in [1] is used for calculations.
- [34] When further speaking about D^+ (D^0) the same about D^- (\bar{D}^0) should be implied.